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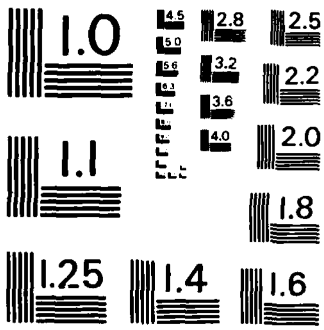


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Professional Paper 430 / January 1985

# THE RELATIONSHIP BETWEEN MATERIAL FAILURES AND FLIGHT HOURS: STATISTICAL CONSIDERATIONS

Matthew S. Goldberg

N00014-83-C-0725

A Division of CNA Hudson Institute

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# THE RELATIONSHIP BETWEEN MATERIAL FAILURES AND FLIGHT HOURS: STATISTICAL CONSIDERATIONS

Matthew S. Goldberg

CNA *Naval Planning, Manpower, and Logistics Division*

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# TABLE OF CONTENTS




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Justification
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**THE RELATIONSHIP BETWEEN MATERIAL FAILURES AND FLIGHT HOURS: STATISTICAL CONSIDERATIONS**

## INTRODUCTION

This paper attempts to clarify the relationships among the following four hypotheses: (1) The number of material failures across intervals of calendar time containing equal accumulated flight hours follows a Poisson distribution; (2) the number of elapsed flight hours between successive independent material failures follows an exponential distribution; (3) the expected number of monthly material failures is exactly proportional to monthly flight hours; and (4) the observed number of monthly material failures is strongly correlated with monthly flight hours.

By a well-known result, hypotheses (1) and (2) are equivalent. By a second well-known result, hypotheses (1) and (3) are also equivalent. However, hypotheses (1) and (4) are not equivalent. That is, while the expected number of failures is exactly proportional to flight hours under a Poisson distribution, this relationship is not revealed by a linear regression between failures and flight hours. For example, we demonstrate that if the mean and variance of flight hours across months are equal and if the failure rate per flight hour equals .01, then the correlation between failures and flight hours will equal only .10. Moreover, the squared correlation, corresponding to the regression R-squared statistic, will equal only .01. Additional keywords: Goodness of fit tests; A-7 aircraft; Naval aircraft.

To test for a relationship between flight hours and the expected number of failures, linear regression is not the appropriate tool. Instead, we must test the underlying hypothesis that the data follow a Poisson distribution. If the Poisson distribution fits the data, then the expected number of failures will be exactly proportional to flight hours because hypothesis (1) implies hypothesis (3) above.

We present three goodness-of-fit tests for a Poisson distribution. It is sometimes asserted that, despite its mathematical tractability, the Poisson distribution does not fit any real-world data. Contrary to this assertion, we show that the Poisson distribution is perfectly adequate to describe data on Navy A-7 accidents over the period CY 1977 – CY 1983.

Use of the Poisson distribution imposes the restriction that the mean and variance of the data are equal. In many situations the negative binomial distribution may provide a superior fit to the data, because the negative binomial distribution allows the variance to exceed the mean. Moreover, the negative binomial distribution may be derived from the Poisson distribution by

assuming that the mean of the Poisson distribution is itself randomly distributed across the observational units according to a Gamma distribution.

Although the negative binomial distribution is more flexible than the Poisson distribution, it is not completely satisfactory because it "explains" the excess variation in the data by simply adding an additional source of randomness. A more useful approach may be to explain the data by introducing observable variables thought to influence the failure rate, such as crew manning and experience levels. The correct technique for estimating the influence of the variables is not linear regression, because linear regression does not even reveal the proportionality between flight hours and the expected number of failures. A better approach is to express the failure rate (rather than the number of failures) as a function of explanatory variables using a maximum likelihood technique known as Poisson regression. We present an exposition of Poisson regression and also some recently developed generalizations of that technique.

## STATISTICAL PRELIMINARIES

The Poisson distribution is frequently used to describe the number of occurrences of an event in a fixed-length interval of time. Let $g(X,t)$ denote the probability of $X$ occurrences in an interval of length $t$. Then the Poisson distribution is given by:

$$g(X, t, \lambda) = (\lambda t)^X exp\,(-\lambda t)/X! \tag{1}$$

for $X = 0, 1, 2, 3...$ where $\lambda > 0$ is the instantaneous failure rate. The Poisson distribution has the property $E(X; \lambda) = Var(X; \lambda) = \lambda t$, so that the mean and variance are constrained to equality.

There are several alternative characterizations of the Poisson distribution. Let $o(t)$ be any function with the property $o(t)/t \rightarrow 0$ as $t \rightarrow 0$. Then

by a well-known result,[1] the following comprise a set of sufficient conditions for the Poisson distribution:

1. $g(1, t) = \lambda t + o(t)$ for all $t > 0$ and some $\lambda > 0$

2. $\sum_{x=2}^{\infty} g(X,t) = o(t)$

3. The numbers of occurrences in any set of non-overlapping intervals are statistically independent.

The first condition states that the probability of exactly one occurrence is proportional to the length of the interval for arbitrarily small intervals. The second condition states that the probability of multiple occurrences approaches zero as the length of the interval approaches zero.

Finally, by another well-known result,[2] the number of occurrences in a fixed-length interval of time has a Poisson distribution if and only if the elapsed time between occurrences has an exponential distribution:

$$f(t) = \lambda\, exp\,(-\lambda t) \quad . \tag{2}$$

## CORRELATION BETWEEN X AND t

As we stated earlier, a property of the Poisson distribution is $E(X; \lambda) = \lambda t$, so that the expected number of occurrences is proportional to the length of the interval. Suppose we have monthly data on material failures ($X$) and flight hours ($t$). If we believe that the observations follow a Poisson distribution, then in view of the proportionality of the mean we might expect a high correlation coefficient between $X$ and $t$. Perhaps surprisingly, this is not the case.

1. See, for example, Hogg and Craig [1], pages 94-96.
2. See, for example, Hogg and Craig [1], pages 100-101.

Note that the density of t across months is arbitrary; we require only that it have finite mean and variance. Let $E_t$ denote the expectation over the density of t. Following the analysis of Brown and Rogers[1] [2]:

$$
\begin{aligned}
E(X) &= E_t E(X|t) \\
&= E_t(\lambda t) \\
&= \lambda E(t).
\end{aligned}
$$

$$
\begin{aligned}
E(Xt) &= E_t E(Xt|t) \\
&= E_t[t\,E(X|t)] \\
&= \lambda E(t^2).
\end{aligned}
$$

$$
\begin{aligned}
E(X^2) &= E_t E(X^2|t) \\
&= E_t[Var(X|t) + E^2(X|t)] \\
&= \lambda E(t) + \lambda^2 E(t^2).
\end{aligned}
$$

Combining these results:

$$
\begin{aligned}
Var(X) &= E(X^2) - E^2(X) \\
&= \lambda E(t) + \lambda^2 [E(t^2) - E^2(t)] \\
&= \lambda E(t) + \lambda^2 \mathrm{Var}(t).
\end{aligned}
$$

$$
\begin{aligned}
Cov(X,t) &= E(Xt) - E(X)E(t) \\
&= \lambda E(t^2) - \lambda E^2(t) \\
&= \lambda Var(t).
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\text{Correlation}\,(X,t) &= [Cov^2(X,t)/Var(X)Var(t)]^{1/2} \\
&= [1 + E(t)/\lambda Var(t)]^{-1/2} \quad . \qquad (3)
\end{aligned}
$$

The correlation will be small as long as $\lambda$ is small and $Var(t)/E(t)$ is not too large. For example, suppose $E(t) = Var(t)$ and $\lambda = .01$. Then the correlation will equal .10. Moreover, the squared correlation, corresponding to the regression R-squared statistic, will equal .01. Hence even if the observations

1. Brown and Rogers actually derive a generalization of what follows in the case of a negative binomial distribution rather than a Poisson distribution.

follow a Poisson distribution exactly, we would not expect a linear regression between $X$ and $t$ to reveal any apparent relationship. The relationship between $E(X)$ and t is exactly proportional, but this relationship is not revealed in the simple scatter plot between $X$ and $t$. Moreover, even if $Var(t)/E(t)$ is large so that the correlation is high, a regression equation will still not provide accurate forecasts of future material failures because future flight hours are highly uncertain and unpredictable in this case.

We may also examine the regression coefficient rather than the correlation coefficient. A linear regression of material failures on flight hours will be unbiased:

$$E(\hat{\lambda}) = Cov(X,t)/Var(t) = \lambda \ .$$

However, the variance of the regression coefficient will be quite large so that the coefficient, although unbiased, will not be precisely estimated. To compute the variance of the regression coefficient, recall the following relationship between the R-squared statistic, t-statistic, and F-statistic from elementary regression theory:[1]

$$(\hat{\lambda})^2/Var(\hat{\lambda}) = t^2 = F = (N-2)R^2/(1-R^2)$$

where $N$ is the sample size. Using equation (3), we find that $Var(\hat{\lambda})$ reduces to:

$$Var(\hat{\lambda}) = [\lambda E(t)]/[(N-2)\,Var(t)] \ .$$

To illustrate the magnitude of $Var(\hat{\lambda})$ again suppose that $E(t) = Var(t)$ and $\lambda = .01$. Then $Var(\hat{\lambda})$ reduces to $.01/(N-2)$. Hence to achieve a "t-ratio" of 2.0, implying statistical significance, the sample size would have to be at least 402. Expressed differently, confidence intervals for $\lambda$ will be extremely wide even for sample sizes of several hundred. Like correlation analysis, regression analysis[2] fails to reveal the proportionality between $E(X)$ and $t$.

1. See, for example, Johnston [3], pages 35-38.
2. The regression may also be computed without an intercept, to reflect the strict proportionality between $E(X)$ and $t$. The regression slope will still be unbiased, the only difference being that $(N-1)$ replaces $(N-2)$ as the degrees-of-freedom in the formulae derived in the text.

To test for a relationship between $E(X)$ and $t$, we must test the underlying hypothesis that the observations follow a Poisson distribution. Examples of such tests are given in the next section.

## TESTS FOR A POISSON DISTRIBUTION

It is sometimes asserted that, despite its mathematical tractability, the Poisson distribution does not fit any real-world data. In this section, we will discuss several tests for a Poisson distribution and provide a counterexample to the above assertion.

As we stated earlier, a property of the Poisson distribution is $E(X;\lambda) = Var(X;\lambda) = \lambda t$. This property provides a test of the Poisson distribution. If the mean and variance are equal, then we accept the Poisson distribution; but if the variance exceeds the mean, we must consider an alternative such as the negative binomial distribution.

We apply this test to data on Class A accidents[1] involving Navy A-7 aircraft over the period CY 1977–CY 1983. Figure 1 plots the monthly number of Class A accidents against the monthly number of A-7 flight hours. Although there does not appear to be any visual relationship, the simple correlation across these 84 monthly data points is actually negative,[2] equal to -.245. However, as we saw in the previous section, this perverse correlation does not invalidate the assumption of a Poisson distibution or the resulting positive proportionality between flight hours and the expected number of accidents.

To test for equality between the mean and variance, we tabulated the number of monthly accidents and then calculated the sample mean and sample variance across the 12 months of each respective year. If the data follow a Poisson distribution, then the mean in each year should equal the variance in that year. The mean and variance may differ from one year to another, but they should move in proportion to each other and to annual flight hours. Hence, if we plot the mean and variance in each year, the seven annual data points should lie along a 45-degree line through the origin.

1. Class A accidents are those which result in either a fatality, complete destruction of an aircraft, or at least $500,000 damage.

2. If the distribution is Poisson, then equation (3) is applicable. Using our sample estimates of E(t), Var(t), and $\lambda$, equation (3) predicts a correlation of +.141. We have not tested whether this discrepancy could reasonably be due to chance.

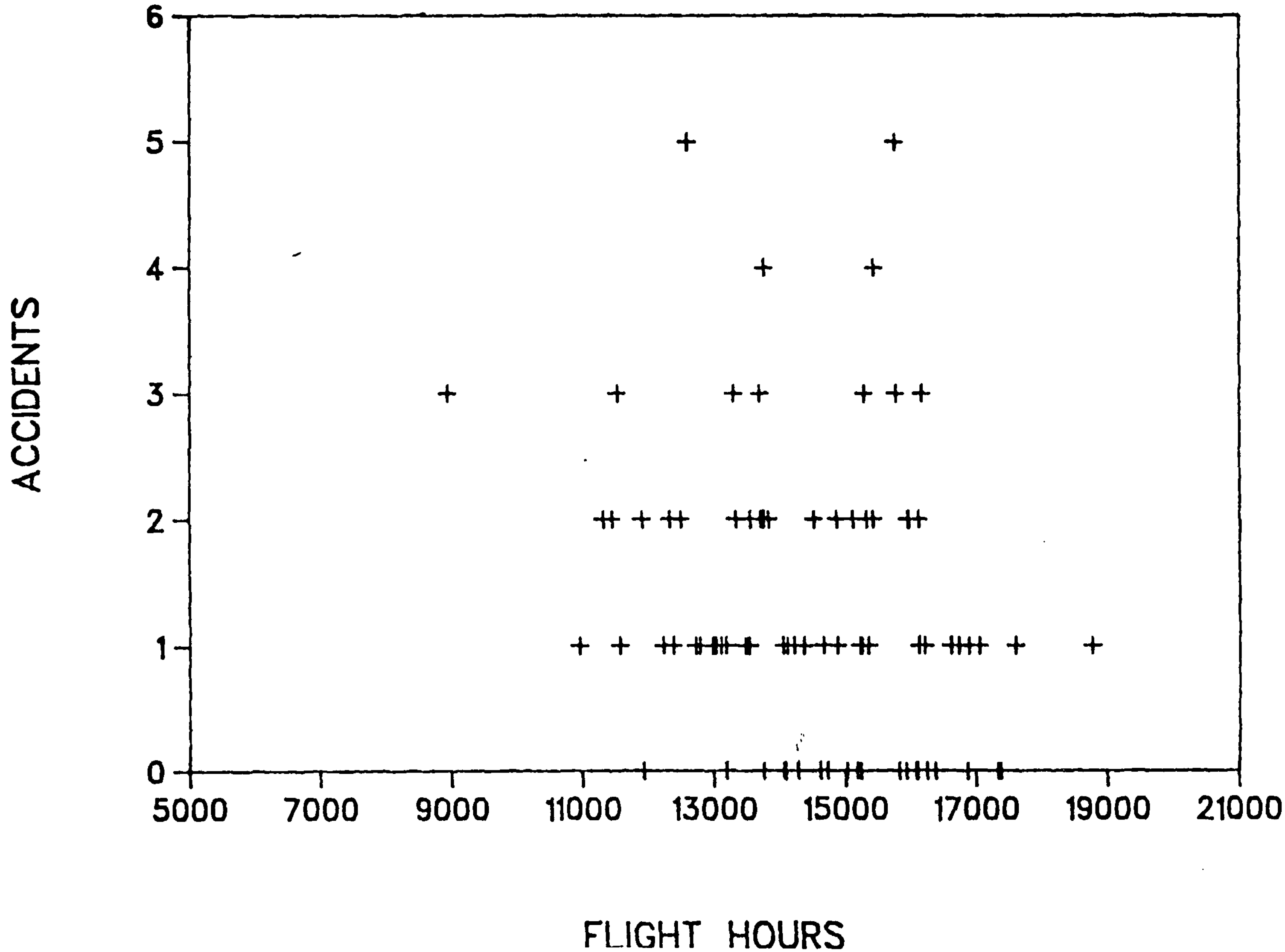


FIGURE 1: MONTHLY FLIGHT HOURS AND ACCIDENTS

Figure 2 presents the plot of the mean and variance. Two of the points lie exactly along the 45-degree line. Four points lie slightly below the line, and one outlier lies well above the line. The outlier is due to the five accidents that occurred during a single month of that year. Overall, the data do not depart radically from the pattern expected under a Poisson distribution.

As a second test, we compared the observed frequency distribution of accidents per period to the best-fitting theoretical Poisson distribution. We could not make this comparison directly using the monthly data because flight hours and hence the Poisson parameter vary across months, so that no single Poisson distribution could possibly fit the entire data set. Instead, to stabilize the Poisson parameter we divided our sample into 81 intervals of calendar time each containing 15,000 accumulated flight hours.[1] We estimated the Poisson parameter for an interval of this length as 15,000 times total accidents divided by total flight hours, yielding the value 1.372. This parameter is quite precisely estimated, with a standard error of only 0.130. The observed distribution and the theoretical distribution with parameter 1.372 are both presented in table 1.

TABLE 1

OBSERVED AND THEORETICAL DISTRIBUTION OF ACCIDENTS[a]

| Number of accidents | Observed frequency | Theoretical frequency |
|---|---|---|
| 0 | 23 | 20.5 |
| 1 | 28 | 28.2 |
| 2 | 16 | 19.3 |
| 3 | 8 | 8.8 |
| 4 | 4 | 3.0 |
| 5 | 1 | 0.8 |
| 6 | 1 | 0.2 |

a. The time interval is 15,000 flight hours, and the corresponding Poisson parameter is 1.372.

1. Actually, the final interval contained only 13,495 flight hours because the total number of flight hours was not evenly divisible by 15,000.

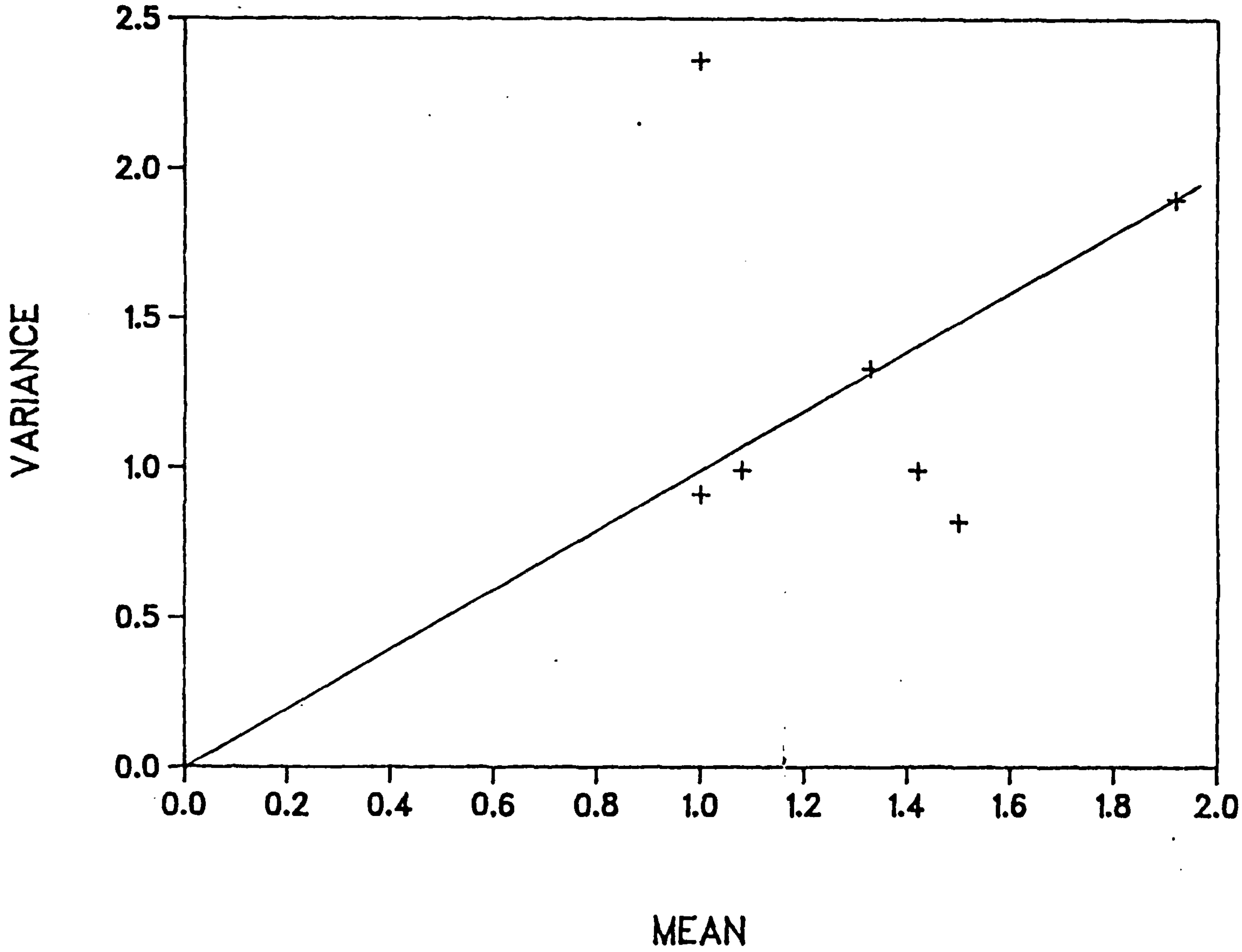


FIGURE 2: MEAN AND VARIANCE OF ANNUAL DATA

We performed a chi-squared goodness-of-fit test to compare the observed and theoretical frequencies. We combined the cells corresponding to 4, 5, and 6 accidents in order to achieve a frequency of at least 4 in every cell. We computed a chi-squared statistic of 1.943 with 3 degrees-of-freedom. This value is much less than the 10 percent significance point of 6.251. Hence we cannot reject the hypothesis of a Poisson distribution.

As a final test, we exploited the fact that the number of accidents per time period has a Poisson distribution if and only if the elapsed flight hours between successive accidents have an exponential distribution. We first computed the elapsed flight hours between successive accidents and placed them in ascending order to obtain the sample order statistics, which we denote $t_1,...,t_n$. If the time between successive accidents, $T$, is exponentially distributed, then the cumulative distribution function at $T$, $Y = F(T) = 1-exp(-\lambda t)$, is uniformly distributed in the interval [0,1]. The i<u>th</u> order statistic in a sample of size n from a uniform distribution has expected value $i/(n+1)$. It follows that:

$$E[1 - exp\ (-\lambda t_i)] = i/(n+1)\ .$$

If we ignore the expectation operator and solve for $t_i$, we obtain:

$$t_i = -(1/\lambda)\ log\ [1 - i/(n+1)]\ . \tag{4}$$

Equation (4) suggests plotting the i<u>th</u> order statistic, $t_i$, against $-log[1-i(n+1)]$. If the distribution is exponential, then all n points should fall along a straight line with slope $1/\lambda$.

Figure 3 contains the plot described above. While we do not perform a formal statistical test, the points indeed seem to fall along a straight line. Moreover, if we fit a least-squares regression line through these points and constrain it to pass through the origin, we obtain a slope coefficient of 11,570.8. Taking the reciprocal of the slope coefficient and multiplying by 15,000, we obtain an estimate of 1.296 as the expected number of accidents in an interval containing 15,000 flight hours. This estimate is quite close to our earlier estimate of 1.372.

Since the Poisson distribution fits the data according to all three goodness-of-fit tests, we conclude that the expected number of accidents is exactly proportional to flight hours. This is true despite the negative simple correlation between monthly accidents and monthly flight hours reported earlier.

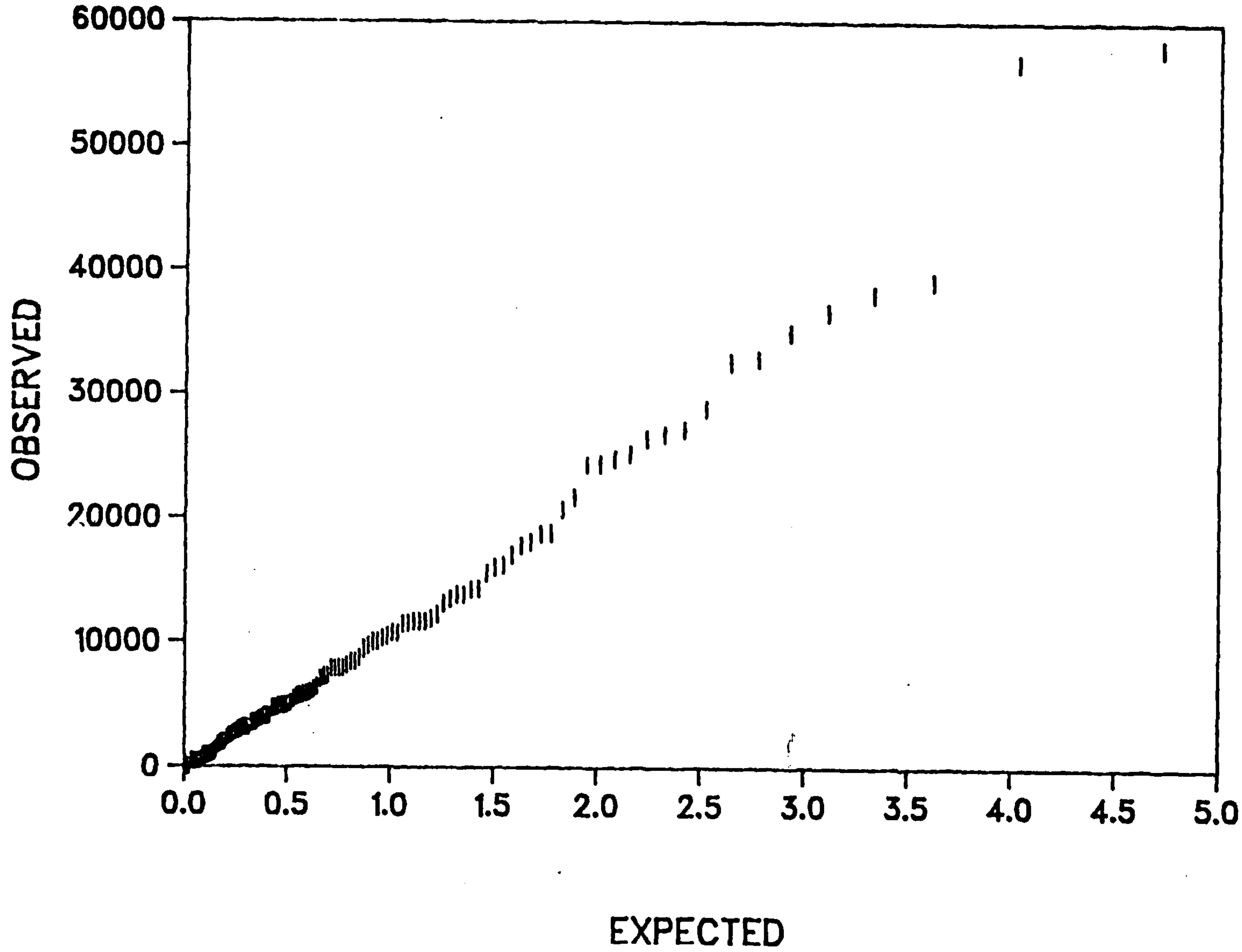


FIGURE 3: EXPECTED AND OBSERVED ORDER STATISTICS

## NEGATIVE BINOMIAL MODEL

Although we have provided an example of real-world data that follow a Poisson distribution, other situations may require a more flexible distribution. In particular, the Poisson distribution constrains the mean and variance to equality. By contrast, the negative binomial distribution allows the variance to exceed the mean.

An interesting derivation of the negative binomial distribution was first given in a classic paper by Greenwood and Yule [4]. Their paper dealt with the distribution of industrial accidents among workers. They assumed that the number of accidents for a given workman in a fixed-length interval of time followed a Poisson distribution. However, they found that different workmen had different degrees of "accident-proneness," hence different means for their respective Poisson distributions. They assumed that the Poisson mean had a Gamma distribution across workmen:

$$h(\lambda) = [\beta^{\alpha} / G(\alpha)]\, \lambda^{\alpha-1}\, exp(-\beta\lambda) \tag{5}$$

where $G$ is the gamma function. Note that the gamma distribution has mean $\alpha/\beta$ and variance $\alpha/\beta^2$. The distribution of accidents across workmen is given by:

$$\int_0^{\infty} g(X, t; \lambda)\, h(\lambda)\, d\lambda$$

where $g(X, t; \lambda)$ is the Poisson distribution in equation (1). After repeated integration by parts, we find that the distribution is equal to:

$$\binom{\alpha+X-1}{X}\left(\frac{t}{\beta+t}\right)^{X}\left(\frac{\beta}{\beta+t}\right)^{\alpha} \tag{6}$$

for $X = 0, 1, 2, 3, \ldots$

Equation (6) is the negative binomial distribution. Its mean is given by:

$$\begin{aligned} E(X) &= E_{\lambda}\, E(X|\lambda) \\ &= E_{\lambda}(\lambda t) \\ &= \alpha t/\beta \end{aligned} \tag{7}$$

To compute the variance, observe that:

$$\begin{aligned} E(X^2) &= E_\lambda E(X^2|\lambda) \\ &= E_\lambda [Var(X|\lambda) + E^2(X|\lambda)] \\ &= \alpha t(\beta + t + \alpha t)/\beta^2 \end{aligned}$$

It follows that:

$$\begin{aligned} Var(X) &= E(X^2) - E^2(X) \\ &= \alpha t(\beta + t)/\beta^2 \end{aligned} \tag{8}$$

Brown and Rogers [2] have applied the same line of reasoning to an analysis of material failures. Moreover, they derived an upper bound on the correlation between $X$ and $t$ that generalizes the expression for the Poisson distribution that we derived earlier in equation (3). Their bound is:

$$\text{Correlation}\,(X,t) < [1 + \beta E(t) / \alpha Var(t)]^{-1/2} \tag{9}$$

Equation (9) is quite analogous to equation (3) because $\alpha/\beta$, the mean of the prior distribution of $\lambda$, replaces the single value of $\lambda$ appearing in equation (3). However, equation (9) is an inequality while equation (3) is an equality. Hence the correlation may be even smaller under a negative binomial distribution than under a Poisson distribution.

We fit a negative binomial distribution to the data on Class A Navy A-7 accidents reported earlier in table 1. Using the method-of-moments, we equated our sample mean of 1.372 to the population mean given by equation (7), and our sample variance of 1.711 to the population variance given by equation (8). We then solved for the values $\alpha = 5.492$ and $\beta = 60{,}154$. These values imply that the prior distribution of $\lambda$ has a mean of .0000913 and a variance that vanishes to 8 decimal places. The negligible variance of the prior distribution implies that the simple Poisson distribution with a fixed value of $\lambda$ seems perfectly adequate to describe the data.

## POISSON REGRESSION MODEL

The negative binomial model provides a flexible tool for situations in which the variance of the data exceeds the mean. However, a drawback of this model is that it "explains" the excess variation in the data by simply adding

an additional source of randomness. A more satisfying approach may be to explain the data by introducing observable variables thought to influence the propensity to fail, such as crew manning and experience levels. The correct technique for estimating the influence of these variables is not a linear regression because, as we have seen, linear regression does not even reveal the proportionality between flight hours and the expected number of failures. A better approach is to express the failure rate (rather than the number of failures) as a function of explanatory variables, using the technique of Poisson regression.

To insure non-negativity, the failure rate in the Poisson regression model is written as an exponential function of the vector $Z_i$ of explanatory variables:

$$\lambda_i = exp\ (Z_i \beta) \tag{10}$$

where $\beta$ is an unknown but estimable vector of coefficients. The likelihood function is obtained by substituting equation (10) into equation (1), and forming the product over all observations in the sample. The vector $\beta$ is chosen to maximize the likelihood function. It is easy to show that the log-likelihood function is globally concave as long as the data matrix (the matrix with rows $Z_i$) is of full rank. Hence the maximum likelihood estimate of $\beta$ is readily obtained using standard algorithms such as Newton's method.

The introduction of explanatory variables will help to eliminate a portion of the excess variation in the data. However, even after controlling for the explanatory variables, the variance of the data may still exceed the mean. To deal with this situation, Hausman, Hall, and Griliches [5] generalized the Poisson regression model to allow random variation in the failure rates beyond the systematic variation induced by the variation in the explanatory variables.

Hausman et al. express the failure rate as:

$$\lambda_i = v_i\, exp\ (Z_i \beta) = exp\, (Z_i \beta + u_i) \tag{11}$$

where $v_i = exp(u_i)$. They assume that $v_i$ has a Gamma distribution across the observational units. Further, they set the two parameters of the Gamma distribution equal, so that the distribution has mean 1 and variance $1/\alpha$ where $\alpha$ is the common value of the two parameters. This normalization involves no loss of generality as long as the data matrix contains a column of 1's corresponding to an intercept.

After repeated integration by parts, we find that the distribution of failures is equal to:

$$\binom{\alpha+X-1}{X}\left(\frac{t\,exp\,(Z_i\beta)}{\alpha+t\,exp\,(Z_i\beta)}\right)^X\left(\frac{\alpha}{\alpha+t\,exp\,(Z_i\beta)}\right)^\alpha \tag{12}$$

for $X = 0, 1, 2, 3...$ Equation (12) is a negative binomial distribution, but its moments are different from those of the negative binomial distribution derived ealier in equation (6). The mean is given by:

$$\begin{aligned} E(X) &= E_v E(X|v) \\ &= E_v[v\,t\,exp\,(Z\beta)] \\ &= t\,exp\,(Z\beta). \end{aligned}$$

To compute the variance, observe that:

$$\begin{aligned} E(X^2) &= E_v E(X^2|v) \\ &= E_v[Var(X|v) + E^2(X|v)] \\ &= t\,exp\,(Z\beta) + t^2[exp\,(Z\beta)]^2(1+\alpha)/\alpha. \end{aligned}$$

It follows that:

$$\begin{aligned} Var(X) &= E(X^2) - E^2(X) \\ &= t\,exp\,(Z\beta) + t^2[exp(Z\beta)]^2/\alpha. \end{aligned}$$

The parameters $\alpha$ and $\beta$ may again be estimated using the method of maximum likelihood. The estimate of $\beta$ obtained from the Poisson regression model provides a convenient starting value for the iteration. However, the log-likelihood function is no longer globally concave, so more care must be exercised in searching for the global maximum.

## CONCLUSIONS

If material failures follow a Poisson distribution, then the expected number of failures is exactly proportional to flight hours. However, this proportionality will not be revealed by simple correlation or regression analysis between monthly flight hours and the number of monthly failures. To test for proportionality, we must test the underlying hypothesis that the data follow a Poisson distribution.

We have presented three goodness-of-fit tests for a Poisson distribution. We have shown that the Poisson distribution is perfectly adequate to describe data on Class A Navy A-7 accidents over the period CY 1977 – CY 1983.

In cases where the Poisson distribution does not fit the data, we present several alternative models. First, the mean of the Poisson distribution may itself be randomly distributed across observational units according to a Gamma distribution. If so, the number of failures will have a negative binomial distribution. Second, the mean of the Poisson distribution may depend systematically upon a set of observable explanatory variables. In this case, the Poisson regression model is appropriate. Finally, the mean of the Poisson distribution may contain both a systematic component that depends upon observable variables and a random component that follows a Gamma distribution. This situation yields a generalized Poisson regression model.

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1. CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311. An index of selected publications is also available on request. The index includes a listing of profesional papers, with abstracts, issued from 1969 to December 1983).
2. Listings for Professional Papers issued prior to PP 407 can be found in *Index of Selected Publications (through December 1983)*, March 1984.

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